

Leggett-Garg tests of macro-realism for multi-particle systems including two-well Bose-Einstein condensates

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We construct quantifiable generalisations of Leggett-Garg tests for macro/ mesoscopic realism and noninvasive measurability that apply when not all outcomes of measurement can be identified as arising from one of two macroscopically distinguishable states. We show how quantum mechanics predicts a negation of the LG premises for proposals involving ideal-negative-result, weak and quantum non-demolition measurements on dynamical entangled systems, as might be realised with two-well Bose-Einstein condensates, path-entangled NOON states and atom interferometers.

Schrodinger raised the apparent inconsistency between macroscopic realism and quantum macroscopic superposition states [1]. Leggett and Garg (LG) suggested to test macroscopic realism against quantum mechanics in an objective sense by comparing the predictions of quantum mechanics with those based on two very general classical premises [2]. The first premise is *macroscopic realism* (MR), that a system which has two macroscopically distinguishable states available to it is at any time in one or other of the states. The second premise is *noninvasive measurability* (NIM), that for such a system it is possible to determine which state the system is in, without interfering with the subsequent evolution of that system.

Leggett and Garg showed how the two premises constrain the dynamics of a two-state system. Considering three successive times $t_3 > t_2 > t_1$, the variable S_i denotes which of the two states the system is in at time t_i , the respective states being denoted by $S_i = +1$ or -1 . The LG premises imply the LG inequality [2, 3]

$$LG = \langle S_1 S_2 \rangle + \langle S_2 S_3 \rangle - \langle S_1 S_3 \rangle \leq 1. \quad (1)$$

and also the “disturbance” or “no signalling in time (NST)” inequality $d_\sigma = \langle S_3 | \hat{M}_2, \sigma \rangle - \langle S_3 | \sigma \rangle = 0$ [4, 5]. Here $\langle S_3 | \hat{M}_2, \sigma \rangle$ (and $\langle S_3 | \sigma \rangle$) is the expectation value of S_3 given that a measurement \hat{M}_2 is performed (or not performed) at time t_2 , conditional on the system being prepared in a state denoted σ at time t_1 . These inequalities can be violated for quantum systems [2–13]. The work of LG represented an advance, since it extended beyond the quantum framework to show how the macroscopic superposition defies classical macroscopic reality.

The LG approach raised new ideas about how to test quantum mechanics even at the microscopic level [6–10]. Failure of the inequalities implies no classical trajectory exists between successive measurements: either the system cannot be viewed as being in a definite state independent of observation, or there cannot be a way to determine that state, without interference by the measurement. Noninvasive measurability would seem “vexing” to justify, however, because of the plausibility of the measurement process disturbing the system. LG countered this problem by proposing an *ideal negative result measurement* (INR): the argument is conditional on the first postulate being true e.g. *if* a photon does travel through

one slit *or* the other, a null detection beyond one slit is justified to be noninvasive [2, 8]. A second approach is to perform *weak measurements* [14, 15] that enable calculation of the moment $\langle S_2 S_3 \rangle$ in a limit where there is a vanishing disturbance to the system [6, 9, 10, 15]. To date, experimental investigations involving INR or weak measurements have focused mostly on microscopic systems e.g. a single photon. An exception is a recent experiment which gives evidence for violation of MR using a simpler form of LG inequality that quantifies the invasiveness of “clumsy” measurements, and is applied to superconducting flux qubits [4]. There have also been recent proposals for LG tests involving macroscopic mechanical oscillators [12] and for macroscopic states of cold atoms, using *quantum non-demolition* (QND) measurements [13].

An illuminating LG test would be for a mesoscopic massive system in a quantum superposition of being at two different locations [16]. An example of such a superposition is the path-entangled NOON state, written as $|\psi\rangle = \frac{1}{\sqrt{2}}\{|N\rangle_a|0\rangle_b + |0\rangle_a|N\rangle_b\}$ where $|N\rangle_{a/b}$ is the N -particle state for a mode a (b) [17]. In this case the ideal negative result measurement can be applied, and justified as noninvasive by the assumption of Bell’s locality [18]. A method is then given to (potentially) negate that the system must be located either “here” or “there”, or else to conclude there is a significant disturbance to a massive system due to a measurement performed at a different location.

In this paper, we show how such tests may be possible on a mesoscopic scale. As one example, we show that LG violations are predicted for Bose-Einstein condensates (BEC) trapped in two separated potential wells of an optical lattice. Here dynamical oscillation of large groups of atoms to form NOON macroscopic superposition states is predicted at high nonlinearities [19–23].

A key problem however for an actual experimental realisation is the fragility of the macroscopic superpositions. To address this problem, we derive *modified LG inequalities* that can be used to test LG premises for superpositions that deviate from the ideal NOON superposition by allowing mode population differences not equal to $-N$ or N . The ideal negative result measurement is difficult to apply where there are residual atoms in both modes, and

we thus develop weak and QND measurement strategies for testing LG premises and demonstrating mesoscopic quantum coherence in this case. Tests of LG realism are also possible for NOON states incident on an interferometer. Finally, we propose a simple LG test for matter waves passing through an atom interferometer could demonstrate the no-classical trajectories result for atoms.

Idealised dynamical two-state oscillation: The Hamiltonian H_I for an N -atom condensate constrained to a double well potential reveals a regime of macroscopic two-state dynamics. The two-well system has been reliably modelled by the Josephson two-mode Hamiltonian [20, 21, 24, 25]:

$$H_I = 2\kappa\hat{J}_z + g\hat{J}_z^2 \quad (2)$$

Here $\hat{J}_z = (a^\dagger a - b^\dagger b)/2$, $\hat{J}_x = (a^\dagger b + b^\dagger a)/2$, $\hat{J}_y = (a^\dagger b - b^\dagger a)/2i$ are the Schwinger spin operators defined in terms of the boson operators \hat{a}^\dagger, \hat{a} and \hat{b}^\dagger, \hat{b} , for the modes describing particles in each of the wells, labelled a and b respectively. The κ models interwell hopping and g the nonlinear self-interaction due to the medium. In a regime of high interaction strength ($Ng/\kappa \gg 1$), a regime exists where if the system is initially prepared with all N atoms in one well, a two-state oscillation can take place with period T_N (Fig. 1) [19, 20]. In one state, $|N\rangle_a|0\rangle_b$, all N atoms are in the well a ($S_i = 1$), and in the second state, $|0\rangle_a|N\rangle_b$, all atoms are in the well b ($S_i = -1$) [20]. If the system is prepared in $|N\rangle_a|0\rangle_b$, then at a later time t' , the state vector is (apart from phase factors)

$$|\psi(t)\rangle = \cos(\tau)|N\rangle|0\rangle + \sin(\tau)|0\rangle|N\rangle \quad (3)$$

where $\tau = E_\Delta t'/\hbar$ and E_Δ is the energy splitting of the energy eigenstates $|N\rangle|0\rangle \pm |0\rangle|N\rangle$ under H_I .

The quantum solution (3) predicts a violation of the LG inequality. The two-time correlation is $\langle S_i S_j \rangle = \cos[2(t_j - t_i)]$ and is independent of the initial state, whether $|N\rangle|0\rangle$ or $|0\rangle|N\rangle$. Choosing $t_1 = 0$, $t_2 = \pi/6$, $t_3 = \pi/3$ (or $t_3 = 5\pi/12$), it is well-known that for this two-time correlation the quantum prediction is $LG = 1.5$ (1.37) which gives a violation of (1) [2].

The tunnelling times in the highly nonlinear regime however are impractically high for proposals based on Rb atoms [26, 27]. The fragility of the macroscopic superposition state will make any such experiment unfeasible [28]. Noting however that the modes a, b of H_I may also describe occupation of two atomic hyperfine levels, κ being the Rabi frequency as in the experiments of [25], the NOON oscillation may well be achievable for other physical realisations of H_I . Alternatively, for more practical oscillation times one can use a different initial state $|N - n_L\rangle|n_L\rangle$, $0 < n_L < N$, where there are atoms in both wells, or else a tilted well [20]. Here, we denote the sign of the spin J_z at time t_i by S_i ($S_i = 1$ if $J_z \geq 0$; $S_i = -1$ if $J_z < 0$). The dynamical solutions presented in Fig. 2 reveal a mesoscopic two-state oscillations over reduced time scales, mimicking the experimentally observations of Albiez et al [27] for $N = 1000$ atoms where oscillations were observed over milliseconds.

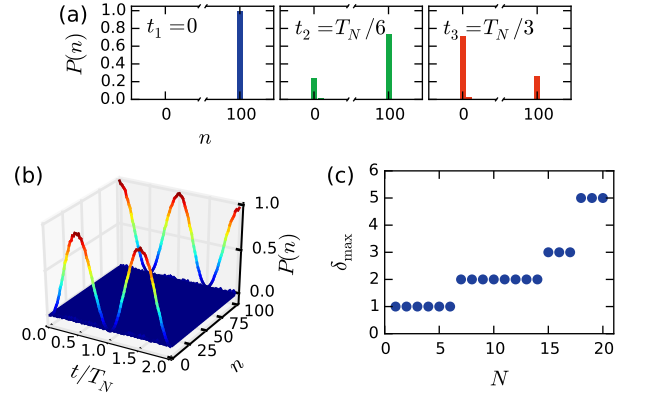


Figure 1. *Oscillating NOON two-state dynamics:* (a) Probabilities of n atoms in mode a at times 0, $T_N/6$, $T_N/3$ and (b) the two-state oscillation for $N = 100$, $g = 1$. (c) Plot gives an upper bound on the backaction δ due to the INR measurement that can be tolerated for an LG violation.

Our objective is to provide practical strategies for testing the LG inequality in such multiparticle experiments. Two questions to be addressed are how to perform (or access the results of) the NIM (assuming it exists), and how to handle the case where the values of S_i may not always correspond to macroscopically distinct outcomes.

To address the first question: As explained in the literature [2], $\langle S_1 S_2 \rangle$ and $\langle S_1 S_3 \rangle$ can be inferred using deterministic state preparation and projective measurements at t_2 and t_3 . To measure $\langle S_1 S_3 \rangle$ no intervening measurement is made at t_2 based on the assumption that the NIM at t_2 will not affect the subsequent statistics. For $\langle S_2 S_3 \rangle$, the evaluation of S_2 is difficult, since with any practical measurement it could be argued that a measurement M made at t_2 is not the NIM, and does indeed influence the subsequent dynamics. Three methods have been used to counter this objection: INR measurements; weak measurements; and quantifiable QND measurements. We next propose LG tests for each case.

(1) *Ideal negative result measurement (INR):* A particularly strong test is possible for experiments involving a NOON superposition (3) where the two modes correspond at time t_2 to spatially separated locations. In this case, the INR strategy similar to that outlined by LG can be applied. A measurement apparatus at time t_2 couples locally to only one mode a , enabling measurement of the particle number n_a . Either $n_a = 0$ or $n_a = N$. Based on the first LG premise, if one obtains the negative result $n_a = 0$, it is assumed that there were prior to the measurement *no* atoms in the mode a . Hence the measurement that gives a negative result is justified to be noninvasive (since $\langle S_2 S_3 \rangle$ can be evaluated using only negative result outcomes [2]). For such an experiment, to assume noninvasive measurability there is implicit the assumption of locality: that there is no change to mode b because of the measurement at a (otherwise a change to the subsequent dynamics could be expected).

Quantification of the NIM premise: We can introduce a quantification of the second LG premise: We suppose

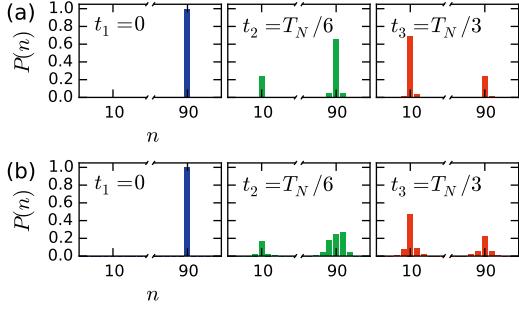


Figure 2. *Mesoscopic two-state oscillations*: Probability of n atoms in mode a at t_2 and t_3 . $N = 100$. The initial state is $n_L = 10$ particles in the right well. Here (a) $g = 3$ and (b) $g = 1$. (a) $LG = 1.49$ using the weak measurement of \hat{J}_z at t_2 and (b) $LG = 1.43$ using the minimal QND measurement of S_2 at t_2 .

that the measurement at mode a (b) can induce a back-action effect on the macroscopic state of the other mode, so that there may be a change of the state of mode b (a) of up to δ particles, where $\delta \leq N$. The change δ may be microscopic, not great enough to switch the system between states $|0\rangle|N\rangle$ and $|N\rangle|0\rangle$, but can alter the subsequent dynamics. The change to the dynamics is finite however, and can be established within quantum mechanics, to give a range of prediction for $\langle S_2 S_3 \rangle$. We have carried out this calculation, and plot the effect of δ for various N in Fig. 1c, noting that a relatively small backaction δ to the quantum state of one mode due to measurement on the other will destroy violations of the LG inequality even for large N .

(2) *Weak and minimally invasive QND measurements*: A second strategy is to construct a measurement that can be shown to give a negligible disturbance to the system being measured. We consider a QND measurement described by the Hamiltonian $H_Q = \hbar G \hat{J}_z \hat{n}_c$. The H_Q models QND measurements of the atomic spin \hat{J}_z based on an ac Stark shift [30]. An optical “meter” field is prepared in a coherent state $|\gamma\rangle$ and coupled to the system for a time τ_0 . The meter field is a single mode with boson operator \hat{c} and number operator $\hat{n}_c = \hat{c}^\dagger \hat{c}$. Writing the state of the system at time t_2 as $\sum_{m=0}^N d_m |m\rangle_a |N-m\rangle_b$ (d_m are probability amplitudes), the output state immediately after measurement is (we set $\tau_0 = \pi/2NG$)

$$|\psi\rangle = \sum_{m=0}^N d_m |m\rangle_a |N-m\rangle_b |\gamma e^{i\pi(N-2m)/2N}\rangle_c \quad (4)$$

Homodyne detection enables measurement of the meter quadrature phase amplitude $\hat{p} = (\hat{c} - \hat{c}^\dagger)/i$. For γ large, the measurement is “strong”, or projective, and the different values of \hat{J}_z (and hence S_2) are precisely measurable as distinct regions of outcomes for \hat{p} . Ideally, a *minimal* QND measurement of S_2 is devised, that does not discriminate the different values of \hat{p} apart from the sign, and leaves all states (4) with definite sign S_2 unchanged.

First, we discuss the *weak* measurement limit, attained as $\gamma \rightarrow 0$. In this limit, we see from (4) that the state of the system is *minimally disturbed* by the measurement. The cost is no clear resolution of the value S_2 for any single measurement. Yet, using (4), we show in the Supplemental Materials that if the system at time t_2 is in a NOON state $d_0|N-n_L\rangle_a|n_L\rangle_b + d_N|n_L\rangle_a|N-n_L\rangle_b$, then

$$\langle S_2 S_3 \rangle = -\frac{1}{2\gamma} \langle p S_3 \rangle \quad (5)$$

where $\langle S_2 S_3 \rangle$ is the value obtained by the projective measurement. Thus, for arbitrarily small γ , the value $\langle S_2 S_3 \rangle$ can be obtained by averaging over many trials. The weak measurement strategy enables a convincing test of the LG premises, since one can experimentally demonstrate the noninvasiveness of the weak measurement, by showing the invariance of $\langle S_1 S_3 \rangle$ as $\gamma \rightarrow 0$ when the measurement is performed at t_2 .

The weak measurement relation (5) does not hold for all input states. However, the *minimal* (“non-clumsy”) QND measurement of S gives a strategy for LG tests, based on extra assumptions. For systems such as in Fig. 2, the state at time t_2 is a superposition of states $|\psi_+\rangle$ and $|\psi_-\rangle$ that give, respectively, outcomes $S = \pm 1$. The first LG premise is that the system is *either* in a state of positive S *or* in a state with negative S . The minimal QND strategy requires a second set of measurements, in order to experimentally establish that states with definite value of S are unchanged by the QND measurement [4, 13, 31]. The noninvasiveness of the measurement is then justified by the first LG premise. We note that in cases where the measurement is not ideal (“clumsy”), the amount of disturbance can be measured and accounted for in a modified inequality as discussed in the Refs. [4, 13, 31]. Strictly speaking, the QND approach is limited to testing a modified LG assumption that the system is always in a *quantum* state with definite S at the time t_2 . This is because it is difficult to prove that all hidden variable states with definite outcome of S are not changed by the QND measurement. Regardless, the approach rigorously demonstrates the *quantum coherence* between the states $|\psi_+\rangle$ and $|\psi_-\rangle$. Fig. 2 shows LG violations using weak and QND measurements.

The s-scopic LG inequalities: We now address how to test macroscopic realism where the system deviates from the ideal of two macroscopically distinguishable states. This occurs when there is a nonzero probability for J_z different to $\pm N/2$ as in Figure 3b. Adapting the approach put forward by LG and Refs. [32], we define three regions of J_z : region “1”, $J_z < -s/2$; region “0”, $-s/2 \leq J_z \leq s/2$; and region “2”, $J_z > s/2$.

For arbitrary s , the MR assumption is accordingly renamed, to *s-scopic realism* (sR). In the generalised case, the meaning of sR is that the system is in a probabilistic mixture of two *overlapping* states: the first that gives outcomes in regions “1” or “0” (denoted by hidden variable $\tilde{S} = -1$); the second that gives outcomes in regions “0” or “2” (denoted by $\tilde{S} = 1$). The second LG premise is

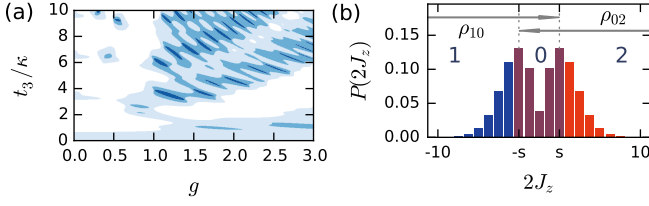


Figure 3. *Violation of s -scopic LG inequalities using nonlinear interferometers:* (a) The NOON state (3) is created at time t_2 ($N = 5$, $\tau = \pi/6$) and evolves to time t_3 according to H_I with nonlinearity g . Contours show regimes for violation of the s -scopic inequality (6) where (dark to light) $s = 4, 2, 0$. (b) Schematic of the probability distribution for results $2J_z$ at t_3 depicting three regions 1, 0, and 2.

generalised to *s -scopic noninvasive measurability* which asserts that such a measurement can be made at time t_2 without changing the result J_z at time t_3 by an amount s or more.

The s -scopic LG premises imply a quantifiable inequality. This is because any effects due to the overlapping region are limited by the finite probability of observing a result there. Defining the measurable marginal probabilities of obtaining a result in region $j \in \{0, 1, 2\}$ at the time t_k by $P_j^{(k)}$, the s -scopic premises are violated if [29]

$$LG_s = P_2^{(2)} - P_1^{(2)} + \langle S_2 S_3 \rangle - (P_2^{(3)} - P_1^{(3)}) - 2P_{0|M}^{(3)} - P_0^{(3)} > 1 \quad (6)$$

where we have used that the system is prepared initially in region 2 and here we restrict to scenarios satisfying $P_0^{(2)} = 0$. The $\langle S_2 S_3 \rangle$ is to be measured using a noninvasive measurement at t_2 . A similar modification is given for the disturbance inequality: The sR premises are violated if $d_{\sigma,s} = |P_{2|M}^{(3)} - P_{1|M}^{(3)} - (P_2^{(3)} - P_1^{(3)})| - (P_0^{(3)} + P_{0|M}^{(3)}) > 0$ where $P_{j|M}^{(3)}$ ($P_j^{(3)}$) is the probability with (without) the measurement M performed at t_2 .

Nonlinear and linear interferometers: Figure 3a shows predictions for s -scopic violations. A NOON state (3) is created at t_2 and a weak measurement or INR performed. The NOON state might be created via the nonlinear H_I or by the conditional methods that have been applied to photonic states [33]. Subsequently, the system evolves according to the nonlinear interaction H_I and a measurement is made of J_z at t_3 . For realistic timescales, there is a spread of J_z at the times t_3 (Fig. 3b). These regimes are realisable for finite g and $N \sim 100$ in BEC nonlinear interferometers [25].

LG tests with mesoscopic superposition/ NOON states are also possible *without nonlinearity* at $t > t_2$, if, after the weak/ INR measurement at t_2 , the two modes are combined across a variable-angle beam splitter (or beam splitter with phase shift ϕ) and J_z of the outputs measured (Fig. 4). The macroscopic Hong-Ou-Mandel technique conditions on $|J_z| > \Delta/2$ ($\Delta < N$) to create at t_2 an N -atom mesoscopic superposition state $|\psi_\Delta\rangle = |\psi_+\rangle + |\psi_-\rangle$ where states $|\psi_\pm\rangle$ are distinct by

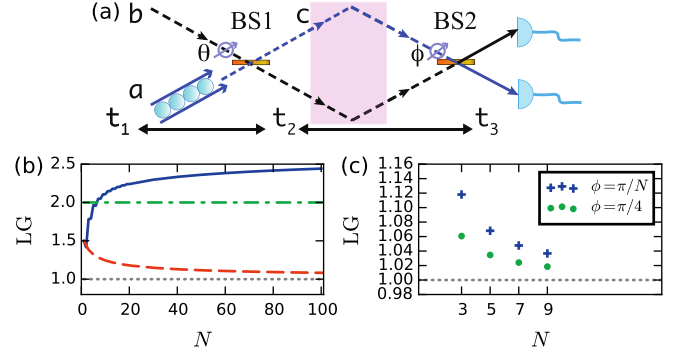


Figure 4. *Multiparticle linear interferometers:* (a) N bosons pass through an interferometer. A QND measurement \hat{M} (purple shading) is made on the state created at t_2 . (b) (blue solid curve) Violation of LG inequality for optimal rotation angles θ, ϕ where \hat{M} measures the number of particles in arm c , showing “no classical trajectories” for individual atoms. (b) (dashed curves) LG violations when \hat{M} is a minimal (non-clumsy) measurement of S_2 . Green dotted-dashed curve shows the disturbance $d_\sigma = 2$ (for all $\Delta < N$) where mesoscopic superposition states $|\psi_\Delta\rangle$ are created at t_2 . Red dashed curve shows LG value for odd N where $\Delta = 0$. For $\Delta = N - 1$, the NOON state (3) with $\tau = \theta$ is created at t_2 . \hat{M} can then be realised as an INR or weak measurement. Fig (c) shows violations in that case using a phase shift ϕ and a 50/50 BS2 (with optimal θ).

more than Δ particles in each arm of the interferometer [33]. Violations of the disturbance and LG inequalities are plotted in Figs. 4b and c. Results indicate small violations for $s \sim 2$ over a range of N and Δ [29].

No-classical trajectories for atom interferometers: Finally, we propose a simple test to falsify classical trajectories in the multi-particle case for simple interferometers. At t_1 , N particles pass through a polariser beam splitter (or equivalent) (BS1) rotated at angle θ (Fig. 4). The number difference of the outputs if measured indicate the value of J_θ (and S_2) at t_2 . The particles are then incident on a second beam splitter BS2 at angle ϕ whose output number difference gives S_3 at t_3 . We invoke the premise, that the system is always in a state of definite J_θ prior to measurement at t_2 . This is based on the hypothesis that *each* atom goes one way *or* the other, through the paths of the interferometer. A second premise is also invoked, that a measurement could be performed of J_θ at t_2 that does not disturb the subsequent evolution. The second premise is justified by the first, and can be supported by experiments that create a spin eigenstate, and then demonstrate the complete invariance of the state after the QND number measurement. If the premises are valid, the LG inequalities (1) will hold, but by contrast are predicted violated by quantum mechanics (Fig. 4b (blue solid curve)). While not the macroscopic test LG envisaged, this gives an avenue for workable tests of the “classical trajectories” hypothesis that could be applied to atoms [34].

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- [1] E. Schroedinger, *Naturwiss.* **23**, 807 (1935).
- [2] A. J. Leggett and A. Garg, *Phys. Rev. Lett.* **54**, 857 (1985).
- [3] A. N. Jordan, A. N. Korotkov, and M. Buttiker, *Phys. Rev. Lett.* **97**, 026805 (2006).
- [4] G. C. Knee et al., *Nat. Commun.* **7**, 13253 (2016).
- [5] Li et al. *Sci. Rep.* **2**, 885 (2012). George et al., *PNAS* **110**, 3777 (2013). Maroney and Timpson, (2014), arXiv:1412.6139v1. J. Kofler and C. Brukner, *Phys. Rev. A* **87**, 052115 (2013). L. Clemente and J. Kofler, *Phys. Rev. A* **91**, 062103 (2015). L. Clemente and J. Kofler, *Phys. Rev. Lett.* **116**, 150401 (2016).
- [6] A. Palacios-Laloy et al., *Nature Phys.* **6**, 442 (2010). C. Emary, N. Lambert and F. Nori. *Rep. Prog. Phys.* **77**, 016001 (2014). J. Dressel et al., *Phys. Rev. Lett.* **106**, 040402 (2011). J. S. Xu et al., *Scientific Report* **1**, 101 (2011). V. Athalye, S. S. Roy, and T. S. Mahesh, *Phys. Rev. Lett.* **107**, 130402 (2011). A. M. Souza, I. S. Oliveira and R. S. Sarthour, *New J. Phys.* **13** 053023 (2011). H. Katiyar et al., *Phys. Rev. A* **87**, 052102 (2013). R. E. George et al., *Proc. Natl Acad. Sci.* **110** 3777 (2013).
- [7] G. Waldbherr et al., *Phys. Rev. Lett.* **107**, 090401 (2011). Z.-Q. Zhou et al., *Phys. Rev. Lett.* **115**, 113002 (2015).
- [8] G. C. Knee et al., *Nature Commun.* **3**, 606 (2012).
- [9] M. E. Goggin, et al., *Proc. Natl. Acad. Sci.* **108**, 1256 (2011).
- [10] Y. Suzuki, M. Iinuma and H. F. Hofmann, *New J. Phys.* **14**, 103022 (2012). J. P. Groen et al., *Phys. Rev. Lett.* **111** 090506 (2013).
- [11] N. Lambert et al., *Phys. Rev. A* **94**, 012105 (2016).
- [12] A. Asadian, C. Brukner and P. Rabl, *Phys. Rev. Lett.* **112**, 190402 (2014).
- [13] C. Budroni et al., *Phys. Rev. Lett.* **115**, 200403 (2015).
- [14] Y. Aharonov, D. Albert and L. Vaidmann, *Phys. Rev. Lett.* **60**, 1351 (1988). N. S. Williams, A. N. Jordan, *Phys. Rev. Lett.* **100**, 026804 (2008).
- [15] J. Dressel et al., *Rev. Mod. Phys.* **86**, 307 (2014). T. C. White et al., *NPJ Quantum Inf.* **2**, 15022 (2016). B. L. Higgins et al., *Phys. Rev. A* **91**, 012113 (2015).
- [16] G. C. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D* **34**, 470 (1986). L. Diósi, *Phys. Lett. A* **120**, 377 (1987). R. Penrose, *Gen. Relativ. Gravit.* **28**, 581 (1996).
- [17] J. P. Dowling, *Contemporary Physics* **49**, 125 (2008). I. Afek, O. Ambar, Y. Silberberg, *Science* **328**, 879 (2010). M. W. Mitchell, J. S. Lundeen and A. M. Steinberg, *Nature* **429**, 161 (2004). P. Walther et al., *Nature* **429**, 158 (2004).
- [18] J. S. Bell, *Physics* **1**, 195 (1964).
- [19] G. J. Milburn et al., *Phys. Rev. A* **55**, 4318 (1997). J. I. Cirac et al., *Phys. Rev. A* **57**, 1208 (1998). J. Dunningham and K. Burnett, *Journ. Modern Optics*, **48**, 1837, (2001). D. Gordon and C. M. Savage, *Phys. Rev. A* **59**, 4623, (1999). Y. Zhou et al., *Phys. Rev. A* **67**, 043606 (2003). T. J. Haigh, A. J. Ferris, and M. K. Olsen, *Opt. Commun.* **283**, 3540 (2010).
- [20] L. D. Carr, D. R. Dounas-Frazer, and M. A. Garcia-March, *Europhys. Lett.* **90**, 10005 (2010). D. R. Dounas-Frazer, A. M. Hermundstad and L. D. Carr, *Phys. Rev. Lett.* **99**, 200402 (2007).
- [21] J. Esteve, et. al., *Nature* **455**, 1216 (2008).
- [22] Q. Y. He et al., *Phys. Rev. A* **86**, 023626 (2012). B. Opanchuk et al., *Phys. Rev. A* **86**, 023625 (2012).
- [23] D. Alcalá, J. Glick and L. Carr, arXiv:1605.05768v1.
- [24] M. Steel and M. J. Collett, *Phys. Rev. A* **57**, 2920 (1998). H. J. Lipkin, N. Meshkov and A. J. Glick, *Nucl. Phys.* **62** 188 (1965).
- [25] C. Gross et al., *Nature* **464** 1165 (2010). M. F. Riedel et al., *Nature (London)* **464**, 1170 (2010).
- [26] D. Ananikian and T. Bergeman, *Phys. Rev. A* **73**, 013604 (2006).
- [27] M. Albiez et al., *Phys. Rev. Lett.* **95** 010402 (2005).
- [28] M. Egorov, et al., *Phys. Rev. A* **84**, 021605(R) (2011). K. S. Hardman et al., *Phys. Rev. A* **89**, 023626 (2014).
- [29] See Supplementary Materials for full details.
- [30] E. O. Ilo-Okeke and T. Byrnes, *Phys. Rev. Lett.* **112**, 233602 (2014). J. Hope and J. D. Close, *Phys. Rev. Lett.* **93**, 180402 (2004).
- [31] A. Leggett, *Foundations of Physics* **18**, 939 (1988). M. Wilde and A. Mizel, *Foundations of Physics* **42**, 256 (2011).
- [32] E. G. Cavalcanti and M. D. Reid, *Phys. Rev. Lett.* **97**, 170405 (2006); *Phys. Rev. A* **77**, 062108 (2008). C. Marquardt et al., *Phys. Rev. A* **76**, 030101R (2007).
- [33] M. Stobinska et al., *Phys. Rev. A* **86** 063823 (2012). T. Sh. Iskhakov et al., *New J. Phys.* **15**, 093036 (2013). K. Yu et al., *New J. Phys.* **16**, 013025 (2014). G. J. Pryde and A. G. White, *Phys. Rev. A* **68**, 052315 (2003). A. E. B. Nielsen and K. Mølmer, *Phys. Rev. A* **75**, 063803 (2007).
- [34] T. Kovachy et al., *Nature* **528**, 530 (2015). K. S. Hardman et al., *Opt. Lett.* **41**, 2505 (2016).